

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

1[30B70, 40A15, 65D15].—LISA LORENTZEN & HAAKON WAADELAND, *Continued Fractions with Applications*, Studies in Computational Mathematics, Vol. 3, North-Holland, Amsterdam, 1992, xvi+606 pp., 24 $\frac{1}{2}$ cm. Price \$157.00/Dfl.275.00.

There are three classic monographs on continued fractions. They are by H. S. Wall [5], O. Perron, [3, 4], and W. B. Jones and W. J. Thron [2]. There is surprisingly little overlap amongst the three with their different styles and emphasis. We now have a fourth, which is again very different. It falls somewhere between textbook and research monograph.

As the authors explain in the Preface, this book is not intended as a substitute for, or an updating of, any of the other three but, rather, as a companion. It is aimed at those in or near mathematics on the one hand, and senior or graduate level students on the other, who are curious about continued fractions.

The style is, for the most part, casual with many motivating and illustrative examples, both theoretical and numerical. A minimum of background is required to appreciate the material, and each but one of the twelve chapters concludes with some Problems, Remarks, and References. The problems are not overly difficult and emphasize the principles of the text. All this makes the book particularly useful as an undergraduate and graduate introductory text. However, even the expert will derive enjoyment and obtain new information and insights from its reading. I certainly did.

The original title of the book "A Taste of Continued Fractions" was perhaps more appropriate since, although the material is indeed tastefully presented, there are instances where the reader is given only the flavor of the subject and is instructed to go elsewhere for a fuller, more satisfying understanding. This is particularly true of Chapters VII, VIII, and IX. A serious omission in connection with Chapter VII is the lack of any reference to the recent developments in hypergeometric orthogonal polynomials, inspired by the work of R. Askey and J. Wilson [1]. Indeed, the book has no hypergeometric examples beyond ${}_2F_1$ or ${}_2\phi_1$. Also missing is the connection with the spectral theory of Jacobi matrices.

Chapter I contains motivating examples and three classical convergence theorems (Sleszynski-Pringsheim, van Vleck, and Worpitzky). Chapter II deals with basic notation and transformations. Chapters III, IV, and V are the main entrée, dealing with Convergence (Stern-Stolz, value sets, parabola and oval theorems,

limit periodic cases), Three-term recurrences (Pincherle's and Auric's theorems and generalizations), and Correspondence, respectively. Chapters VI, VII, and VIII give a taste of the connections with hypergeometric functions, moments and orthogonality, and Padé approximants, respectively. The final four chapters have applications to number theory, zero-free regions of polynomials, digital filter theory, and differential equations. The Appendix is an (admittedly incomplete) catalogue of known continued fraction expansions. Although there are some references at the end of each chapter, there is, unfortunately, no overall bibliography. A Subject Index completes the text.

The book is recommended as a tasteful introduction to continued fractions, which will stimulate an appetite for further reading and prepare one for digesting current research on the subject.

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1. R. Askey and J. Wilson, *Some basic hypergeometric orthogonal polynomials that generalize Jacobi polynomials*, Mem. Amer. Math. Soc. **319** (1985).
2. W. B. Jones and W. J. Thron, *Continued fractions: Analytic theory and applications*, Addison-Wesley, Reading, Mass., 1980.
3. O. Perron, *Die Lehre von den Kettenbrüchen, Band I: Elementare Kettenbrüche*, Teubner, Stuttgart, 1954.
4. O. Perron, *Die Lehre von den Kettenbrüchen, Band II: Analytisch-funktionentheoretische Kettenbrüche*, Teubner, Stuttgart, 1957.
5. H. S. Wall, *Analytic theory of continued fractions*, Van Nostrand, New York, 1948.

2[01-00, 11A55, 30B70, 40A15, 41A21, 65B10].—CLAUDE BREZINSKI, *History of Continued Fractions and Padé Approximants*, Springer Series in Computational Mathematics, Vol. 12, Springer-Verlag, Berlin, 1991, 551 pp., 24 cm. Price \$79.00.

As the author admits in the Introduction to this book, he realized soon after having embarked on the project of writing a history of continued fractions that he had neither the time nor the inclination to write a complete history of the subject. Thus he restricted himself to presenting "a collection of facts and references about continued fractions." Moreover, "this history ends with the first part of our century, that is, 1939."

The contents of the book are fairly well described by the titles of the sections of the book, which are as follows:

1. Euclid's algorithm, the square root, indeterminate equations, history of notations.
2. Ascending continued fractions, the birth of continued fractions, miscellaneous contributions, Pell's equation.
3. Brouncker and Wallis, Huygens, number theory.
4. Euler, Lambert, Lagrange, miscellaneous contributions, the birth of Padé approximants.
5. Arithmetical continued fractions, algebraic properties, arithmetic, applications, number theory, convergence, algebraic continued fractions, expansion